Importance Sampling for McKean-Vlasov SDEs

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Abstract

This work deals with the Monte-Carlo methods for evaluating expectations of functionals of solutions to McKean-Vlasov Stochastic Differential Equations (MV-SDE) with drifts of super-linear growth. We assume that the MV-SDE is approximated in the standard manner by means of an interacting particle system and propose two importance sampling (IS) techniques to reduce the variance of the resulting Monte Carlo estimator. In the {complete measure change} approach, the IS measure change is applied simultaneously in the coefficients and in the expectation to be evaluated. In the {decoupling} approach we first estimate the law of the solution in a first set of simulations without measure change and then perform a second set of simulations under the importance sampling measure using the approximate solution law computed in the first step.

For both approaches, we use large deviations techniques to identify an optimisation problem for the candidate measure change. The decoupling approach yields a far simpler optimisation problem than the complete measure change, however, we can reduce the complexity of the complete measure change through some symmetry arguments. We implement both algorithms for two examples coming from the Kuramoto model from statistical physics and show that the variance of the importance sampling schemes is up to 3 orders of magnitude smaller than that of the standard Monte Carlo. The computational cost is approximately the same as for standard Monte Carlo for the complete measure change and only increases by a factor of 2–3 for the decoupled approach. We also estimate the propagation of chaos error and find that this is dominated by the statistical error by one order of magnitude.

Keywords: McKean, Vlasov Stochastic Differential Equation, interacting particle system, Monte Carlo simulation, importance sampling, large deviations

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